Written exam for the M. Sc. in Economics June 3, 2010

Economic Growth

Master's Course

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is: Problem 1 25%, Problem 2 65%, and Problem 3 10%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 A famous paper by Mankiw, Romer, and Weil (1992) carries out a cross-country regression analysis (98 countries, 1960-1985) based on the aggregate production function,

$$Y_t = K_t^{\alpha} H_t^{\beta} (T_t L_t)^{1-\alpha-\beta}, \qquad 0 < \alpha < 1,\ 0 < \beta < 1, \tag{*}$$

where Y is GDP, K aggregate capital input, H aggregate human capital input, T the technology level, and L input of man hours. The gross investment rates in the two types of capital are a fraction s_K and s_H of GDP, respectively. Assuming $T_t = T_0 e^{gt}$, $g \geq 0$, is the same for all countries in the sample (apart from a noise term affecting T_0), the authors conclude that $\alpha = \beta = 1/3$ fits the data well.

Let h denote average human capital, i.e., $h \equiv H/L$, and suppose all workers at any time t have the same amount of human capital, equal to h_t.

a) Show that (*) can be rewritten on the form $Y_t = F(K_t, X_t L_t)$ and indicate what X_t must be.

b) When we study individual firms' decisions, this alternative way of writing the production function is more convenient than the form (*). Explain why.

c) Within a Ramsey-style set-up, where s_K and s_H are endogenous and time-dependent, it can be shown that the economy converges to a steady state with $\tilde{y} \equiv Y/(TL) = (\tilde{k}^*)^{\alpha} (\tilde{h}^*)^{\beta}$, where $\tilde{k}^*$ and $\tilde{h}^*$ are the constant steady state values of $\tilde{k} \equiv K/(TL)$ and $\tilde{h} \equiv h/T$. Find the long-run growth rate of $y \equiv Y/L$. Does human capital accumulation drive per capita growth in the long run?

Section 5.1 in the textbook by Barro & Sala-i-Martin also uses a Ramsey-style one-sector approach to human and physical capital accumulation. Their production function is

$$Y_t = K_t^{\alpha} (h_t L_t)^{1-\alpha}. \tag{**}$$

We shall compare the implications of (*) and (**) under the assumption that T_t in (*) is time-independent and equals 1.

d) Does (*) and (**) imply the same or different answers to the last question in c)? Comment.

e) Briefly evaluate the Barro & Sala-i-Martin set-up from a theoretical as well as empirical perspective.

Problem 2 Consider the "simple increasing variety model" with permanent monopolies. Firm i ($i = 1, 2, ..., M$) in the competitive manufacturing sector has the production function

$$Y_i = A \left(\sum_{j=1}^{N} {x_{ij}}^{\alpha} \right) L_i^{1-\alpha}, \qquad A > 0,\ 0 < \alpha < 1. \tag{1}$$

Here Y_i, L_i, and x_{ij} denote output of the firm, labor input, and input of intermediate good j, respectively ($j = 1, 2, ..., N$; N "large"). The model is in continuous time, but the time subscript on the variables is implicit when not needed for clarity. The labor force is $L = \sum_i L_i$ and is constant.

a) The symmetry in (1) and the fact that the prices of intermediate goods are all set (by monopoly firms) at the same level $P = 1/\alpha$ induce firm i to choose the same $x_{ij} = x_i$ for all j. Explain by a few well-chosen sentences why this is so. Next derive the implied result:

$$Y_i = AN{x_i}^{\alpha} L_i^{1-\alpha}. \tag{2}$$

b) Increasing variety models are inspired by the hypothesis that "variety is productive" or, with a broader formulation, "there are gains by specialization". Can the link to this hypothesis be visualized by rewriting equation (2)? Comment.

c) The market mechanism ensures that all firms in the manufacturing sector choose the same x_i/L_i ratio. Explain why this is so and show that $y_i \equiv Y_i/L_i$ will be the same for all i.

An implication of x_i/L_i being independent of i is that we can write

$$\frac{x_i}{L_i} = \frac{X_m}{L}. \tag{3}$$

d) Interpret X_m and explain how (3) comes about.

The aggregate production function in manufacturing at time t can be written

$$Y_t = AX_t^{\alpha}(N_t L)^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1, \tag{4}$$

where $X_t = N_t X_m = N_t(\alpha^2 A)^{1/(1-\alpha)} L$. This aggregate manufacturing output is used partly for replacing the intermediate goods used up in the production of Y_t, partly for consumption, C_t, and partly for investment in R&D, R_t :

$$Y_t = X_t + C_t + R_t. \tag{5}$$

The invention production function is

$$\dot{N}_t \equiv \frac{dN_t}{dt} = \frac{R_t}{\eta}, \qquad \eta > 0, \ \eta \text{ constant}. \tag{6}$$

e) Derive (4). *Hint:* $Y_t = \sum_i y_{it} L_{it}$.

f) Given the "growth engine", as specified by (4), (5), and (6) taken together, is the model likely to be technologically capable of generating fully endogenous growth? Why or why not?

Let the household sector consist of L households with infinite horizon, a pure rate of time preference, ρ, and a CRRA instantaneous utility function with parameter $\theta > 0$ (only consumption enters the utility function).

g) Write down the implied Keynes-Ramsey rule for the individual household.

h) Determine the equilibrium real interest rate and the per capita consumption growth rate along an equilibrium path with positive R&D. *Hint:* a certain no-arbitrage condition and the formula $\pi = \left(\frac{1}{\alpha} - 1\right) X_m$ are useful here.

i) Along the equilibrium path determine the growth rate of N, Y, R, and X (a detailed derivation need not be given; instead you may refer to a general property of reduced-form AK models in a Ramsey framework). If one or more parameter restrictions are needed to ensure that an equilibrium path with positive R&D can exist, write it/them down.

We now change the set-up by introducing a government sector. Suppose the government does two things. First, it employs a given constant fraction $s \in (0, 1)$ of the labor force as civil servants, whose services affect neither *marginal* utility of private consumption, nor productivity in the production sectors; the wage paid to civil servants is the same as the wage for other workers in the economy. Second, the government levies a tax on consumption at a constant rate $\tau > 0$ to finance the wage payments to civil servants.

j) Assume that s is of moderate size so that an equilibrium path with positive R&D still exists. Determine the growth rate of c and N along such a path. *Hint:* employment in the manufacturing-goods sector is now $L' = (1 - s)L$.

k) What is the effect of a higher s on growth? Why?

ℓ) We now change the set-up once more by assuming that civil servants provide, first, rule-of-law and social-trust services, and second, technical-scientific services reducing private research costs. What parameters in the model might then depend on s? How may this affect your conclusion in k)?

m) "Whatever the answers in ℓ) are, there is scope for Pareto improvement in the economy." True or false? Why?

Problem 3 *Short questions*

a) "Arrow's learning-by-investing model predicts that the share of capital income in national income is constant in the long run if and only if the aggregate production function is Cobb-Douglas." True or false? Why?

b) In the theory of growth based on endogenous technical change we encounter different hypotheses about the invention production function. List some examples. Briefly comment.
